Chapter 7
The Pilot-Wave Theory

According to the orthodox interpretation of quantum mechanics, the theory provides – already, with wave functions alone – complete descriptions of physical states. In Chaps. 3–5 we reviewed three distinct but inter-related problems that afflict this view, at least according to people like Einstein, Schrödinger, and Bell. The alleged problems could be summarized by saying that there seems to be something deficient about quantum mechanical wave functions as descriptions of physical reality – either (at best) the wave functions provide only an *incomplete* description of what is actually going on physically, or (at worst) they fail to provide any comprehensible description of physically real processes at all.

Bohr and Heisenberg, of course, did not accept the criticisms and built a rather elaborate philosophical edifice in support of the claim that the theory is not only perfectly rational and comprehensible, but indeed complete. Their arguments, however, were never very convincing to the critics. For example, in 1949 (that is, well after the debates that led up to and followed the Schrödinger's Cat and EPR episodes of 1935), Einstein wrote that "the statistical quantum theory does not pretend to describe the individual system (and its development in time) completely". And so, Einstein said, "it appears unavoidable to look elsewhere for a complete description of the individual system...." From the point of view of a theory that *did* "accomplish a complete physical description", "the statistical quantum theory would ... take an approximately analogous position to the statistical mechanics within the framework of classical mechanics [1]."

The current chapter presents a concrete example of a theory of this sort – one which purports to *complete* the usual quantum mechanical description of physical states (by adding something new). The theory was first proposed, but then prematurely abandoned, by de Broglie in the mid 1920s [2]. The theory was then independently rediscovered and further developed by David Bohm in 1952 (and is therefore sometimes called "Bohmian Mechanics") [3]. Bell, who championed the theory until his untimely death in 1990, gave a very nice overview of its basic idea when he wrote:

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> While the founding fathers agonized over the question
>
> ‘particle’ *or* ‘wave’
>
> de Broglie in 1925 proposed the obvious answer
>
> ‘particle’ *and* ‘wave’.
>
> Is it not clear from the smallness of the scintillation on the screen that we have to do with a particle? And is it not clear, from the diffraction and interference patterns, that the motion of the particle is directed by a wave? De Broglie showed in detail how the motion of a particle, passing through just one of two holes in [a] screen, could be influenced by waves propagating through both holes. And so influenced that the particle does not go where the waves cancel out, but is attracted to where they cooperate. This idea seems to me so natural and simple, to resolve the wave-particle dilemma in such a clear and ordinary way, that it is a great mystery to me that it was so generally ignored. Of the founding fathers, only Einstein thought that de Broglie was on the right lines. Discouraged, de Broglie abandoned his picture for many years. He took it up again only when it was rediscovered, and more systematically presented, in 1952, by David Bohm. .... There is no need in this picture to divide the world into ‘quantum’ and ‘classical’ parts. For the necessary ‘classical terms’ are available already for individual particles (their actual positions) and so also for macroscopic assemblies of particles [4].

Let’s try to understand in more detail what this theory says and how it works.

7.1 Overview

According to the de Broglie - Bohm pilot-wave theory, most of the puzzles and paradoxes of orthodox quantum mechanics arise from its using *incomplete* state descriptions. It is not, for example, that electrons are wave-like when not being observed, but then magically “collapse” to sharp positions when looked at. Instead, according to the pilot-wave theory, the electron is always a particle with a sharp position following a definite trajectory through space; the statistical wave-like phenomena (such as the build-up of the interference pattern in the two-slit experiment) arise because the motion of the particle is influenced by an associated wave. This is sometimes hard for people to understand because they are so accustomed, in ordinary quantum mechanics, to describing “particles” (like electrons) in terms of wave functions. So let me repeat it for emphasis: a single electron, according to the pilot-wave theory, is not one thing, but *two* – a wave *and* a (literal, pointlike) particle whose motion is controlled by the wave.

To formulate the theory in a rigorous way, we need to know the dynamical laws obeyed by both the wave and the particle. For the wave this is easy, because the wave is nothing but the ordinary quantum mechanical wave function Ψ obeying Schrödinger’s equation:

$$i\hbar\frac{\partial\Psi}{\partial t} = \hat{H}\Psi. \tag{7.1}$$

So that part is familiar and straightforward.

But what about the motion of the particle? Here we can take a clue from the de Broglie formula

$$p = \frac{h}{\lambda} = \hbar k \tag{7.2}$$

which relates the momentum p of the particle with the wavelength λ (or wave number k) of the associated wave. (Note that this equation is very difficult to understand unless there genuinely exist two things: a wave *and* a particle!) This suggests that when the wave function is a plane-wave $\Psi \sim e^{ikx}$ with a definite wave number k, the particle should move with velocity

$$v = \frac{p}{m} = \frac{\hbar}{m}k. \tag{7.3}$$

But what should the velocity be in the general case, where the wave function is not of this very special plane-wave type, and hence has no single well-defined wave number k?

The following seems like the simplest way of generalizing the last equation. Write the wave function in "polar form" $\Psi(x,t) = R(x,t)e^{iS(x,t)}$ (so that R is the modulus and S is the *phase* of the wave function) and then let

$$v = \frac{\hbar}{m}\frac{\partial S}{\partial x}. \tag{7.4}$$

For the plane-wave type solution, $S(x,t) = kx$ and so Eq. (7.4) reduces to Eq. (7.3). But Eq. (7.4) makes sense in general, for any $\Psi(x,t)$. Well, except for one thing: for a generic wave function $\Psi(x,t)$, the gradient of the phase $(\partial S/\partial x)$ will be a function of x and t. So where, exactly, should we evaluate the function to give the velocity of the particle? The obvious answer is: evaluate it at the actual location $X(t)$ of the particle! We will thus consider the following as the simplest possible candidate law describing how the particle moves under the influence of the wave:

$$\frac{dX(t)}{dt} = \frac{\hbar}{m}\left.\frac{\partial S(x,t)}{\partial x}\right|_{x=X(t)} \tag{7.5}$$

where $S(x,t)$ is the complex phase of the wave function. Note that this can be equivalently re-written (in terms of the wave function itself) as follows:

$$\frac{dX(t)}{dt} = \frac{\hbar}{m}\,\mathrm{Im}\left[\frac{\left(\frac{\partial\Psi}{\partial x}\right)}{\Psi}\right]\Bigg|_{x=X(t)}. \tag{7.6}$$

where "Im" means "the imaginary part".

That is basically all there is to the theory: a single electron (for example) is a wave *and* a particle, with the wave just obeying Schrödinger's equation and the particle moving, under the influence of the wave, according to Eq. (7.6). There is one other

aspect of the theory, though, that we will develop and explain here even though, in some sense, it is less fundamental. This has to do with how probabilities arise and are understood and explained in the theory.

Let's begin by reviewing/recalling an important fact about Schrödinger's equation, here, for simplicity, for a single particle moving in one dimension:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x,t)\Psi. \tag{7.7}$$

The complex conjugate of Schrödinger's equation reads:

$$-i\hbar\frac{\partial\Psi^*}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi^*}{\partial x^2} + V(x,t)\Psi^*. \tag{7.8}$$

(Note that we assume here that the potential energy $V(x,t)$ is real.) Now multiply Eq. (7.7) by Ψ^*, multiply Eq. (7.8) by Ψ, and subtract the second from the first. The result is

$$i\hbar\left[\Psi^*\frac{\partial}{\partial t}\Psi + \Psi\frac{\partial}{\partial t}\Psi^*\right] = -\frac{\hbar^2}{2m}\left[\Psi^*\frac{\partial^2}{\partial x^2}\Psi - \Psi\frac{\partial^2}{\partial x^2}\Psi^*\right] \tag{7.9}$$

which can be simplified to

$$\frac{\partial}{\partial t}|\Psi|^2 = -\frac{\partial}{\partial x}\left[\frac{i\hbar}{2m}\left(\Psi\frac{\partial}{\partial x}\Psi^* - \Psi^*\frac{\partial}{\partial x}\Psi\right)\right]. \tag{7.10}$$

This has the form of the so-called "continuity equation"

$$\frac{\partial\rho}{\partial t} = -\frac{\partial}{\partial x}j \tag{7.11}$$

or, in three dimensions,

$$\frac{\partial\rho}{\partial t} = -\vec{\nabla}\cdot\vec{j}. \tag{7.12}$$

This continuity equation is satisfied, for example, in electrodynamics by the electric charge density ρ and the electric current density $\vec{j}$. In this context, we think of the continuity equation as expressing the local conservation of charge: a positive divergence of $\vec{j}$ at some point, which implies a net outward flow of electric charge away from that point, corresponds to a negative $\frac{\partial\rho}{\partial t}$, i.e., a decreasing charge density at the point.

So in the same way, Eq. (7.10) can be understood as expressing the local conservation of *probability* because we recognize $|\Psi|^2$ as the standard expression for the probability density (of finding the particle, if we look for it) in quantum mechanics. We thus identify

$$j = \frac{i\hbar}{2m}\left(\Psi\frac{\partial}{\partial x}\Psi^* - \Psi^*\frac{\partial}{\partial x}\Psi\right) \tag{7.13}$$

or, in three dimensions,

$$\vec{j} = \frac{i\hbar}{2m}\left(\Psi\vec{\nabla}\Psi^* - \Psi^*\vec{\nabla}\Psi\right) \tag{7.14}$$

as the "quantum probability current".

The fact that Schrödinger's equation implies that $|\Psi|^2$ obeys the continuity equation (with the $\vec{j}$ just given) is a completely standard (if slightly advanced) principle of orthodox quantum mechanics which has nothing in particular to do with the pilot-wave theory. But it relates to the pilot-wave theory in two (related!) ways.

First, recall that in electrodynamics the electrical current density (associated with, say, a single charged particle) is just the charge density multiplied by the particle's velocity: $\vec{j} = \rho\vec{v}$. And so one can write the velocity as the ratio of the current and charge densities like this:

$$\vec{v} = \frac{\vec{j}}{\rho}. \tag{7.15}$$

Now, in orthodox quantum mechanics, we have a probability density $\rho = |\Psi|^2$ and probability current $\vec{j}$. Of course, in *orthodox* quantum mechanics, there are no (literal) particles, but only wave functions. But – given orthodox quantum mechanics – if you wanted to propose that, in addition to the wave function, there is also a (literal) particle, it would be very natural and obvious – based on the analogy with electrodynamics – to guess that the velocity might be given by

$$\vec{v} = \frac{\vec{j}}{\rho} = \frac{i\hbar}{2m}\frac{\Psi\vec{\nabla}\Psi^* - \Psi^*\vec{\nabla}\Psi}{\Psi^*\Psi} \tag{7.16}$$

or, switching back to one dimension,

$$v = \frac{j}{\rho} = \frac{i\hbar}{2m}\frac{\Psi\frac{\partial}{\partial x}\Psi^* - \Psi^*\frac{\partial}{\partial x}\Psi}{\Psi^*\Psi}. \tag{7.17}$$

But it is now easy to see that this is yet another way of re-writing the equation, that we guessed above, for the velocity of the particle in the pilot-wave theory: since

$$\frac{\left(\Psi\frac{\partial}{\partial x}\Psi^* - \Psi^*\frac{\partial}{\partial x}\Psi\right)}{-2i} = \mathrm{Im}\left(\Psi^*\frac{\partial}{\partial x}\Psi\right) \tag{7.18}$$

Equation (7.17) becomes

$$v = \frac{\hbar}{m}\frac{\mathrm{Im}\left(\Psi^*\frac{\partial}{\partial x}\Psi\right)}{\Psi^*\Psi} = \frac{\hbar}{m}\mathrm{Im}\left[\frac{\left(\frac{\partial\Psi}{\partial x}\right)}{\Psi}\right] \tag{7.19}$$

which is the same as Eq. (7.6).

So that is the first reason for going into the quantum continuity equation here: it gives another illuminating perspective on the pilot-wave theory's new dynamical postulate (for how the particles should move).

The second reason is that it makes it possible to understand something about probability in the theory. In general, according to the pilot-wave theory, every particle is always definitely somewhere. But we do not usually know the exact location! Indeed, if we experimentally prepare an electron to have wave function $\Psi(x, 0)$, we cannot pick or control the exact initial particle position $X(0)$ – this will therefore be *random*, and one might expect that there should be some associated probability distribution $P(x, 0)$ to characterize this. But since we have already committed to a specific formula for the velocity that particles at various positions x and times t would have, it is clear that the initial probability distribution $P(x, 0)$ will change in time. It is possible to show that the probability distribution $P(x, t)$ should satisfy

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial x}\left[\, v(x,t)\, P(x,t)\,\right] \tag{7.20}$$

(see the Projects). But then one can see that $P(x, t) = |\Psi(x, t)|^2$ is a special, equilibrium probability distribution for the pilot-wave theory, in the following sense: if $P(x, 0) = |\Psi(x, 0)|^2$ at the initial time ($t = 0$), then we will have $P(x, t) = |\Psi(x, t)|^2$ for all times. This is often described in the literature by saying that the distribution $P = |\Psi|^2$ is "equivariant". The proof of this is just the following: if $P = |\Psi|^2$ and $v = j/|\Psi|^2$, then Eq. (7.20) reduces to the continuity equation, Eq. (7.10), which we already showed is satisfied as a consequence of Schrödinger's equation.

There is a lot more that can be said about how to understand the quantum probabilities in the pilot-wave theory. One of the theory's main virtues is that something like the Born rule can be genuinely *derived* (from the basic dynamical postulates of the theory) rather than merely posited as an additional axiom. What we have just been explaining is a part (but only a part) of that derivation, but it would be too big a distraction to go any deeper into this. So for our purposes here it will have to suffice to think about the theory in something like the following way: at some cosmological initial time $t = 0$, the wave function was Ψ_0 and the particle positions were selected, randomly, according to the $|\Psi|^2$ distribution at that initial time. It then follows, from the two dynamical postulates of the theory, that we – today, inside the universe – should see particle positions that are distributed according to the Born rule: $P(x, t) = |\Psi(x, t)|^2$.

We will illustrate these ideas with a concrete example in the following section.

7.2 Particle in a Box

Let's try to see more clearly how the pilot-wave theory works by considering the simple example of a (one-dimensional) particle-in-a-box (PIB). Suppose to begin with that the system is in the ground state so that

$$\Psi(x,t) = \psi_1(x)e^{-iE_1t/\hbar}. \tag{7.21}$$

Since $\psi_1(x) = \sqrt{\frac{2}{L}}\sin(\pi x/L)$ is purely real, the complex phase associated with Ψ is just

$$S(x,t) = -iE_1t/\hbar. \tag{7.22}$$

This doesn't depend on x at all, so the particle velocity, according to Eq. (7.5), is *zero*. The particle, that is, just sits there at rest. This, as it turns out, is characteristic of so-called stationary states, which are indeed aptly named according to this theory.

Note that this applies also to some more interesting and realistic situations: for example, the electron in a Hydrogen atom in its ground state is not, according to the pilot-wave theory, orbiting the proton, but is instead just sitting there, at some fixed point near the proton. If that bothers you or seems physically impossible, you are probably tacitly expecting that if the electron is literally a particle, it should obey Newton's equations of motion, and should therefore accelerate toward the proton due to the electrostatic force. But the pilot-wave theory is not classical mechanics! The motion of the particle, according to this theory, is not determined by classical forces acting on it, but is instead determined by the structure of the wave function which guides it.

To see some non-trivial dynamics in the particle-in-a-box system, we need only let the quantum state be a superposition of energy eigenstates. For example, suppose the wave function is given by

$$\begin{aligned}\Psi(x,t) &= \frac{1}{\sqrt{2}}\left[\psi_1(x)e^{-iE_1t/\hbar} + \psi_2(x)e^{-iE_2t/\hbar}\right] \\ &= \frac{1}{\sqrt{L}}\left[\sin(\pi x/L)e^{-i\omega_1 t} + \sin(2\pi x/L)e^{-i\omega_2 t}\right].\end{aligned} \tag{7.23}$$

It is slightly cumbersome here to put this in polar form, but straightforward to use Eq. (7.6) to express the particle velocity as a function of its position:

$$\frac{dX(t)}{dt} = \frac{\hbar}{m}\mathrm{Im}\left[\frac{\frac{\pi}{L}\cos\left(\frac{\pi x}{L}\right)e^{-i\omega_1 t} + \frac{2\pi}{L}\cos\left(\frac{2\pi x}{L}\right)e^{-i\omega_2 t}}{\sin\left(\frac{\pi x}{L}\right)e^{-i\omega_1 t} + \sin\left(\frac{2\pi x}{L}\right)e^{-i\omega_2 t}}\right]_{x=X(t)}. \tag{7.24}$$

This is a bit of a messy first-order differential equation, but it's easy enough to let Mathematica solve it numerically. See Fig. 7.1 for some example world lines.

Basically, what happens is that – as the wave intensity sloshes back and forth within the box (as we saw in Chap. 2) – the particle is pushed back and forth with it. The ensemble of trajectories in Fig. 7.1 is, however, a little uneven and funny-looking because we have chosen an ensemble in which the initial positions $X(0)$ are equally spaced, i.e., the initial distribution $P(x)$ is constant. One can see in the figure that the distribution then becomes very non-constant (with several trajectories bunching closely together) after a short period of time.

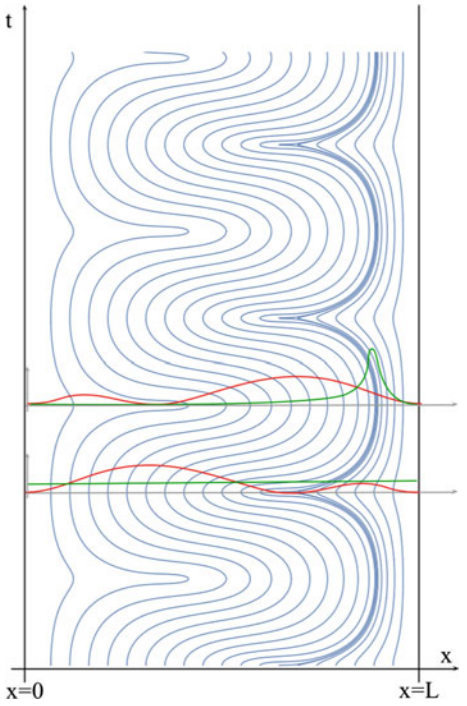

Fig. 7.1 The *blue curves* are a set of possible worldlines for a particle-in-a-box with wave function an equally-weighted superposition of the $n = 1$ and $n = 2$ energy eigenstates. (This is a space-time diagram, with the horizontal axis being the position x within the box, and the vertical axis representing the time t.) Note that at $t = 0$ the example trajectories are equally-spaced across the box. If we think of this as an ensemble of trajectories, we would say that the initial distribution $P(x)$ is constant. But then the distribution at later times is not constant, as illustrated by the rather extreme "clumping up" of the world lines. The distribution P is graphed, as a function of x, at two different times: see the *green curves* that live on the grey axes, whose vertical location is meant to indicate the time. The associated *red curves* show what $|\Psi|^2$ looks like at these same times

As described in the last section, however, there is a special distribution whose functional form is preserved in time. This is the distribution $P = |\Psi|^2$ in which the number of trajectories in the ensemble is proportional to the "intensity" $|\Psi|^2$ of (i.e., what is in orthodox QM thought of as the "probability density" associated with) the wave function Ψ. The claim, then, is that if we have an ensemble of particles (all moving under the influence of the same wave function $\Psi(x, t)$) with, at $t = 0$, the distribution $P(x, 0) = |\Psi(x, 0)|^2$, then it follows from Eqs. (7.1) and (7.6) that $P(x, t) = |\Psi(x, t)|^2$ for all t. As mentioned before, this property is sometimes called the "equivariance" of the $|\Psi|^2$ distribution.

The equivariance property is illustrated in Fig. 7.2, which is the same as Fig. 7.1 except that now the distribution of initial positions $X(0)$ is given by $|\Psi(x, 0)|^2$. One can see that, indeed, the distribution continues to be given by $|\Psi(x, t)|^2$ for later times.

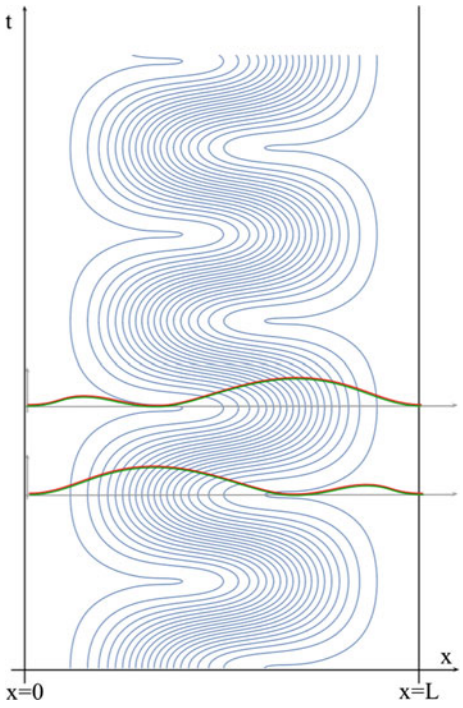

Fig. 7.2 Same as Fig. 7.1 but for an ensemble of initial positions $X(0)$ that are distributed with $P(x,0) = |\Psi(x,0)|^2$. This illustrates the "equivariance" property discussed in the previous section: if the positions of particles in the ensemble are $|\Psi|^2$-distributed at $t = 0$, then they will remain $|\Psi|^2$-distributed for all time. So the *green curves*(indicating P) and the *red curves*(indicating $|\Psi|^2$) coincide at all times here, unlike the situation depicted in the previous figure

So, this example illustrates all of the main ideas of the pilot-wave theory: a quantum system is a hybrid of particle-and-wave, with the wave being simply the usual wave function obeying Schrödinger's equation. The particle has a random initial position within the wave, and this position then evolves in time according to the guidance equation (which we have written in several mathematically equivalent forms). The motion of the particle is indeed well-captured by Bell's statement that the particle "is attracted to where [the contributions to Ψ] cooperate", i.e., where there is constructive interference, i.e., where $|\Psi|^2$ is large. For example, here, at $t = 0$ $|\Psi|^2$ is large on the left side of the box and small on the right, but after a short period of time $|\Psi|^2$ becomes small on the left and large on the right; the particles thus move from left to right to "follow" $|\Psi|^2$.

7.3 Other Single Particle Examples

Let's review a couple of other examples to get a sense of how the theory works. Consider, to start, the spreading Gaussian wave packet from Chap. 2. We saw that if, at $t = 0$, the wave function is given by

$$\Psi(x,0) = Ne^{-x^2/4\sigma^2} \tag{7.25}$$

then

$$\Psi(x,t) = N(t)\, e^{-x^2/4(\sigma^2 + i\hbar t/2m)}. \tag{7.26}$$

where $N(t)$ is a time- (but not position-) dependent complex normalization constant. We can put this in "polar form" by multiplying (inside the argument of the exponential) by the complex conjugate of $(\sigma^2 + i\hbar t/2m)$ divided by itself. This gives

$$\Psi(x,t) = N(t)\, \exp\left[\frac{-x^2\sigma^2}{4(\sigma^4 + \hbar^2 t^2/4m^2)}\right] \exp\left[\frac{ix^2\hbar t}{8m(\sigma^4 + \hbar^2 t^2/4m^2)}\right]. \tag{7.27}$$

So we can identify the complex phase $S(x,t)$ of the wave function as[1]

$$S(x,t) = \frac{x^2\hbar t}{8m(\sigma^4 + \hbar^2 t^2/4m^2)}. \tag{7.28}$$

Plugging this into Eq. (7.5) gives the following first-order differential equation for the position $X(t)$ of a particle being guided by this spreading Gaussian packet:

$$\frac{dX(t)}{dt} = X(t)\, \frac{t}{t^2 + 4m^2\sigma^4/\hbar^2}. \tag{7.29}$$

It is not hard to show that this differential equation is solved by

$$X(t) = X_0 \left(1 + \frac{t^2}{4m^2\sigma^4/\hbar^2}\right)^{1/2} \tag{7.30}$$

which can be re-written as

$$\left(\frac{X(t)}{X_0}\right)^2 - \left(\frac{t}{2m\sigma^2/\hbar}\right)^2 = 1. \tag{7.31}$$

This is the equation for a *hyperbola*, and so it turns out that the spreading Gaussian wave packet has the nice feature that the world lines of particles are hyperbolae. Some example trajectories are shown in Fig. 7.3.

To summarize this first example, when an initially narrow wave packet *spreads*, as of course occurs for example in the phenomenon we call *diffraction*, according to the pilot-wave theory the possible particle trajectories also spread out from one another, as one would expect on the basis of the equivariance property.

[1] Technically, there is also a contribution to the complex phase from what I called $N(t)$, but since that only depends on time, and we ultimately only care about the derivative of $S(x,t)$ with respect to x, I am just ignoring that other contribution. What's written here, then, is really just the x-dependent part of $S(x,t)$.

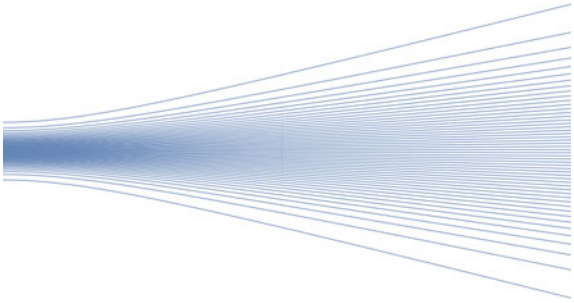

Fig. 7.3 Representative sample of particle trajectories for a spreading Gaussian wave packet. Here time runs to the *right* and x runs vertically (so it is a space-time diagram turned sideways). Or one can, in the spirit of Fig. 2.6, consider replacing t with a second spatial coordinate, and hence think of the lines as showing the trajectories through space that particles would follow downstream of a single Gaussian slit. That is, the figure can be understood as showing the trajectories that particles would follow when being guided by a diffracting wave function. Note that the distribution of initial particle positions $X(0)$ here is given by $|\Psi(x,0)^2|$, so (by "equivariance") the trajectories spread out from one another so as to keep $P = |\Psi|^2$ for subsequent times

We can use a similar technique to visualize the possible particle trajectories in the case of two-slit interference. Beginning with a superposition of two Gaussian wave packets (centered at $x = a$ and $x = -a$) we showed already in Chap. 2, Eq. (2.57), that

$$\Psi(x,t) = N(t)\left[e^{-\frac{(x-a)^2}{4(\sigma^2+i\hbar t/2m)}} + e^{-\frac{(x+a)^2}{4(\sigma^2+i\hbar t/2m)}}\right]. \tag{7.32}$$

This is a little harder to write explicitly in "polar" form (although there is a reasonably simple way of writing $S(x,t)$ explicitly). And the differential equation one gets for $X(t)$ has nothing as simple as hyperbolae as solutions. The only hope, really, is to solve the differential equation for $X(t)$ numerically using a computer. See Fig. 7.4 for the beautiful results!

There are a number of other simple example scenarios for which it is illuminating to consider the particle trajectories as posited by the pilot-wave theory. See, for example, the Projects for references to two papers which analyze (i) a simple case of reflection and transmission at a potential step and the case of quantum tunneling through a classically-forbidden region and (ii) the pilot-wave theory's account of *spin* and its measurement in for example a Stern–Gerlach type apparatus. But hopefully the examples discussed above already give you a fairly clear sense of how the theory works in simple situations. So let us then turn to exploring some other important aspects of the theory.

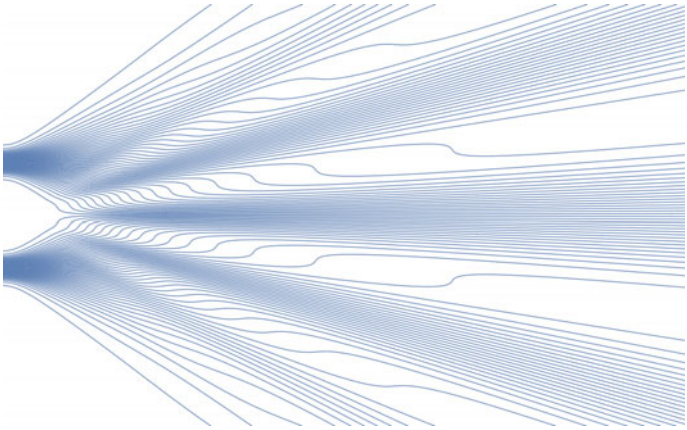

Fig. 7.4 Representative sample of particle trajectories for the case of two initially-separated Gaussian wave-packets. As in the previous figure, this is technically a space-time diagram turned sideways – but one may also legitimately think of it as showing the trajectories, through space, of particles which have just emerged, moving to the *right*, through a double- (Gaussian) slit screen. This type of image, of the particle trajectories for the double-slit experiment according to the pilot-wave theory, was first presented in Ref. [5] and has become iconic for the pilot-wave theory because it captures so clearly, in a picture, how the theory explains the (otherwise) puzzling wave-particle-duality. The discrete flashes on the detection screen correspond to places where (literal, pointlike) particles collide with the screen; but the highly non-classical motion of the particles is influenced by the accompanying pilot-wave such that the particle trajectories bunch up around points of constructive interference. An ensemble of such trajectories (with suitably random initial conditions) will therefore perfectly account (in, to use Bell's phrase, "a clear and ordinary way" [4]) for the type of statistical interference pattern we saw in Fig. 2.8

7.4 Measurement

We assumed, in our discussion of the one-particle examples above, that if we make a position measurement at some time t when the wave function is $\Psi(x, t)$ and the actual particle position is $X(t)$, we will see the particle where it *is*. For example, if a particular particle in the double slit experiment is following one of the trajectories shown in Fig. 7.4, we will see a "flash" on the detection screen right where the particle runs into it, i.e., at the location where the trajectory it's following intersects the screen (on, for example, the right of the figure).

But probably the most important virtue of the pilot-wave theory is that we do not need to divide up the world into "quantum system" (which we describe using the theory) and "classical environment" (which we take for granted and make uncontrolled assumptions about) in order to understand measurements and their outcomes. Instead, we are free (indeed, required!) to enlarge the "quantum system" (which we describe using the theory) until it includes literally everything – the entire universe. This is of course in contrast to ordinary quantum mechanics which, as we discussed in detail in Chap. 3, seems to require one to introduce what Bell called a "shifty

split" (i.e., an artificial division of the world into distinct "quantum" and "classical" realms, with special ad hoc exceptions to the usual dynamical rules when the two realms interact). The claim, then, is that unlike orthodox quantum mechanics, the pilot-wave theory is not afflicted with a "measurement problem."

Let us discuss this in terms of the simple example, from Chap. 3, in which the energy of a particle-in-a-box (with degree of freedom x) is measured, and the outcome displayed in the position of a "pointer" (with degree of freedom y). As discussed back in that chapter, a schematic interaction Hamiltonian

$$\hat{H}_{int} = \lambda \hat{H}_x \hat{p}_y \tag{7.33}$$

can be shown to generate the expected kind of behavior, namely, that if the initial wave function is given by

$$\Psi(x, y, 0) = \psi_n(x)\phi(y) \tag{7.34}$$

(where $\psi_n(x)$ is the nth energy eigenstate of the particle-in-a-box and $\phi(y)$ is a gaussian wave packet centered at $y = 0$, the "ready" position of the pointer), Schrödinger's equation will give the time-evolved wave function

$$\Psi(x, y, t) = \psi_n(x)\phi(y - \lambda E_n t) \tag{7.35}$$

in which the wave packet for the pointer has *moved* a distance proportional to the energy E_n of the particle in the box. In short, the post-interaction position of the pointer registers the actual energy E_n of the particle-in-the-box... just as it should if the process is going to be described as a measurement of that energy!

The trouble arose when we considered what happens if the particle-in-a-box starts out in a superposition of different energy eigenstates. From the linearity of Schrödinger's equation, it is clear that if

$$\Psi(x, y, 0) = \left[\sum_i c_i \psi_i(x)\right] \phi(y) \tag{7.36}$$

then we will have

$$\Psi(x, y, t) = \sum_i c_i \psi_i(x)\phi(y - \lambda E_i t). \tag{7.37}$$

That is, instead of having a well-defined post-interaction position which we can interpret as registering the (single, well-defined) outcome of the measurement, the pointer itself becomes "infected" with the quantum superposition. That is, the final state is a superposition of terms like: "the particle is in the ground state and the pointer indicates $E = E_1$", but also "the particle is in the first excited state and the pointer indicates $E = E_2$", and so on. The wave function alone fails to pick out a particular result; instead it contains, so to speak, all possible results in parallel. But since in an

actual measurement of this kind we always *observe* a single, definite result, it seems that the wave function alone is inadequate to account for our observations. That, in a nutshell, was the measurement problem.

How does the pilot-wave theory resolve the problem, given that, as we have said, the theory *also* says that there is a wave function which obeys Schrödinger's equation? It is true that, according to the pilot-wave theory, the PIB-pointer system has a wave function that ends up in the state described by Eq. (7.37). But the crucial idea is that, according to the pilot-wave theory, *the wave function alone does not provide a complete description of the physical situation*. There is, in addition, the actual position $X(t)$ of the PIB and – crucially here – the actual position $Y(t)$ of the pointer.

Let's think a little bit about what the theory says these actual particle positions do. The details are a little bit complicated (mostly because of the somewhat unusual form of the interaction Hamiltonian) so I'll save those for the Projects at the end of the chapter. But, in principle, the idea is simple: the position $X(t)$ of the particle-in-the-box evolves according to

$$\frac{dX(t)}{dt} = \frac{j_x}{|\Psi|^2} \tag{7.38}$$

where j_x is the x-component of the quantum probability current associated with this system. Similarly, the position $Y(t)$ of the pointer evolves according to

$$\frac{dY(t)}{dt} = \frac{j_y}{|\Psi|^2} \tag{7.39}$$

where j_y is the y-component of the probability current. And at some level you don't really need to worry about what, exactly, the trajectories look like, because you know – from the equivariance property – that if the initial values $X(0)$ and $Y(0)$ start out random (and suitably distributed) the positions will remain $|\Psi|^2$-distributed for all times. And so the positions $X(t)$ and $Y(t)$ later on will be random and they will in particular lie somewhere in the support of $\Psi(x, y, t)$. Let's think qualitatively about what that means.

Figure 7.5 shows a "cartoon" representation of how the wave function $\Psi(x, y, t)$ evolves in time as the measurement interaction proceeds. At $t = 0$, the wave function has support between $x = 0$ and $x = L$ and for $y \approx 0$. But as time goes on, the superposed terms in Ψ move, in the y-direction, by different amounts, so that after a while the wave function has support in a set of discrete, non-overlapping "islands" of the configuration space. But the *particles* have definite positions $X(t)$ and $Y(t)$ which, together, can be understood as defining an "actual configuration point" which traces out some kind of trajectory through configuration space. (I've indicated a possible beginning and end to this trajectory in the figure by putting a dot at $\{X(0), Y(0)\}$ on the left and $\{X(t), Y(t)\}$ on the right.) The exact trajectory will be, in general, rather twisted and complicated: as the several wave function "islands" slide across one another (in the process of going from their initial, stacked configuration, to their final, separated, configuration) there is a complicated interference pattern, somewhat

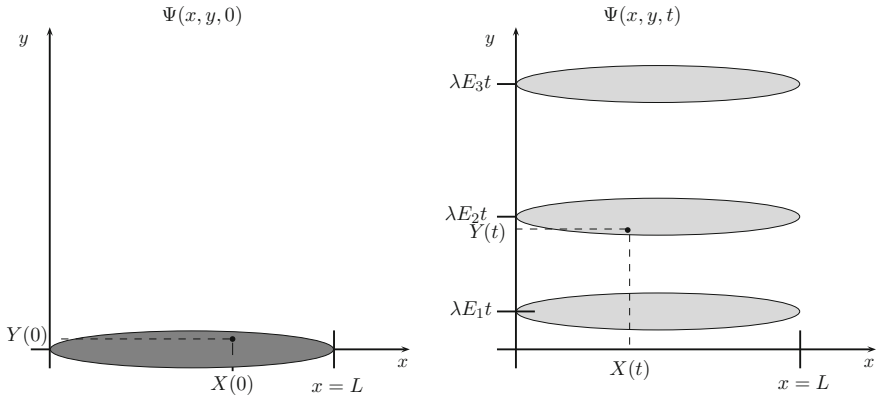

Fig. 7.5 The graph on the *left highlights* (in *dark gray*) the region of the two-dimensional configuration space where $\Psi(x, y, 0)$ has support. Later, at time t, the wave function has split apart into several non-overlapping "islands". This is depicted in the graph on the *right*. The simultaneous presence of all these islands constitutes, for orthodox quantum mechanics, the measurement problem. But for the pilot-wave theory, the actually-realized outcome of the measurement is not to be found in the wave function, but rather in the final position of the pointer. And this, in the pilot-wave theory, will be some one (random but perfectly definite) value, indicated here by the vertical position $Y(t)$ of the *dot* which represents the actual configuration point (X, Y). The indicated $Y(t)$ is in the support of the $n = 2$ branch of the wave function – i.e., $Y(t)$ is approximately $\lambda E_2 t$ – so we would say in this case that the energy measurement had the outcome $E = E_2$. Note that the outcome might have been different had the (random) initial positions $X(0)$ and $Y(0)$ been different

like the one that makes the particle trajectories bend this way and that in the double slit setup. But as the "islands" cease to overlap, things calm down, and the actual configuration point $\{X(t), Y(t)\}$ finds itself in one or the other of the islands. And so, in particular, the pointer has some specific post-interaction position, $Y(t)$ which is either approximately $\lambda E_1 t$, or $\lambda E_2 t$, or $\lambda E_3 t$, etc. So, at the end of the experiment, the position of the pointer is in no way "blurry" or "indefinite" or "superposed". The pointer has a definite position which corresponds to exactly *one* of the energy values we regard as possible outcomes of the experiment.

Indeed, note that, by the equivariance property (and, again, assuming the initial positions $X(0)$ and $Y(0)$ are suitably random), the probability for the final configuration point $\{X(t), Y(t)\}$ to lie in the nth "island" is equal to the integral of $|\Psi|^2$ across that island. But this is simply $|c_n|^2$ – the very thing we would have identified, according to the textbook quantum rules, as the probability for the measurement to have the n^{th} outcome. So the pilot-wave theory reproduces the statistical predictions of ordinary QM. But it does this while treating the system-being-measured and the measuring apparatus itself, on an equal footing, both as part of a big system that is described in a uniform way by the theory.

It is perhaps also worth stressing here that the pilot-wave theory generates the same *statistical* predictions as ordinary QM, even though the theory is completely *deterministic*. Recall that in ordinary QM, the Schrödinger equation part of the dynamics is

completely deterministic; it is the collapse postulate (i.e., the dynamics that momentarily pre-empts the usual Schrödinger evolution when a "measurement" occurs) that introduces the randomness which results in the theory making statistical predictions. In the pilot-wave theory, there is no collapse postulate: the wave function obeys the Schrödinger equation *always*. And the dynamics describing the motion of the particles is also completely deterministic. Randomness enters – like in classical statistical mechanics – only through the initial conditions. Basically, we can never be sure (in advance) how a quantum measurement will come out, because we can never know (in sufficient microscopic detail) what the initial positions of all the particles were.

There is much more that can be said about this issue. It turns out, for example, that the uncertainty (associated with initial particle positions) is *unavoidable*. If, for example, you prepare a system to have wave function $\psi(x)$, then the *best possible knowledge* that you could in principle have, about the position X of the associated particle, is (according to the theory) that $P(x) = |\psi|^2$. Thus, although lots of detailed microscopic structure *exists*, according to the pilot-wave theory – for example, every particle in the universe has at every moment a precisely defined position and velocity – not all of this structure is *accessible* to us. The theory thus allows us to understand Heisenberg's uncertainty principle as just that – uncertainty about facts which exist. This is in contrast to the usual interpretation, in which, to be precise about it, it is not so much that we are *uncertain* about these things, but that the things themselves are objectively *indefinite*.

Let us develop one more point which ties some of these ideas together. We have been stressing that in the pilot-wave theory one can – and indeed must – treat *everything* (ultimately, the whole universe) quantum mechanically. So for example when a measuring apparatus interacts with some particle (one of whose properties is being "measured") we must solve Schrödinger's equation for the entire system comprising both the particle and the apparatus. And as we saw, no additional ad hoc postulates (about "collapse", etc.) are needed to explain how the measurement has a single, definite outcome: although the wave function (for the whole big system) ends up in an entangled superposition, the particles (being, after all, literal particles) have definite positions all the time.

That's great if one just wants to understand why there is no "measurement problem" in the pilot-wave theory. But the broader claim is that the pilot-wave theory reproduces all of the statistical implications of ordinary textbook quantum theory. And ordinary textbook quantum theory generally does not include measuring devices in the systems being described quantum mechanically, but instead just talks about (for example) the PIB whose energy is being measured. And one of the things it says about the PIB is that the PIB has its own wave function (both before and after the measurement) and that, during the measurement, this wave function *collapses*. So this naturally raises the question: even though there is nothing like a collapse *postulate* in the pilot-wave theory, can the theory nevertheless shed some light on the success and utility of the textbook quantum rules?

The answer is yes, and this is one of the most interesting (and also least appreciated) aspects of the theory. To begin with, note that even though there is in some sense really only one big wave function in the pilot-wave theory (namely, the wave

function of the whole universe) the theory allows one to define the wave function for a sub-system. The way to do this is as follows: just evaluate the universal wave function at the actual locations of all the particles *outside* the subsystem. For example, take the case of the PIB-pointer system we've been discussing here. The wave function for the whole PIB-pointer system is

$$\Psi(x, y, t) = \sum_i c_i \psi_i(x) \phi(y - \lambda E_i t). \tag{7.40}$$

By simply evaluating this at $y = Y(t)$ – the actual location of the pointer particle – one thus has a function which depends only on x and t and can be understood as the wave function, call it $\chi(x, t)$ of the PIB sub-system:

$$\chi(x, t) \sim \sum_i c_i \psi_i(x) \phi(Y(t) - \lambda E_i t). \tag{7.41}$$

The "$\sim$" is there, instead of an "$=$" sign, because it would probably be sensible to define the sub-system wave function in such a way that it is properly normalized. The RHS, however, is not. But this is a minor technical detail that we simply leave aside for now.

Here is the important thing. At $t = 0$ (or, in general, before the interaction with the measuring apparatus is turned on), the PIB sub-system wave function is

$$\chi(x, 0) \sim \sum_i c_i \psi_i(x)\, \phi\,(Y(0)) \sim \sum_i c_i \psi_i(x) \tag{7.42}$$

since $\phi\,(Y(0))$ is just a constant that doesn't depend on x or i. This is just what we would ordinarily have said the PIB's pre-measurement wave function is, according to the textbook theory. So that is not too interesting or surprising. But consider what happens for large t (after, say, the measurement has gone to completion). Recall in particular that the actual pointer position $Y(t)$ ends up (at random, depending on the uncontrollable initial conditions) *either* near $\lambda E_1 t$ *or* $\lambda E_2 t$ *or* $\lambda E_3 t$, etc. That is, $Y(t) \approx \lambda E_n t$ for some particular n which we describe as the actual outcome of the experiment. But since ϕ is something like a narrow Gaussian wave packet, this means that $\phi(Y(t) - \lambda E_i t)$ will be approximately zero for all values of i except $i = n$, the one corresponding to the realized outcome. And so this means that, for large t, the PIB sub-system wave function can be written

$$\chi(x, t) \sim \sum_i c_i \psi_i(x) \phi(Y(t) - \lambda E_i t) \approx c_n \psi_n(x) \phi(Y(t) - \lambda E_n t). \tag{7.43}$$

But this is equivalent to saying

$$\chi(x, t) = \psi_n(x) \tag{7.44}$$

since c_n and $\phi\,(Y(t) - \lambda E_n t)$ are again just constants that don't depend on x.

Thus, the wave function of the PIB sub-system evolves, during the course of the interaction with the measuring apparatus, from a superposition of several energy eigenstates, into the one particular eigenstate that corresponds to the realized outcome of the experiment. In fact this evolution is perfectly smooth and continuous; but if the interaction is strong, the evolution will occur rapidly, and one might be forgiven for describing it as apparently discontinuous. The point is, of course, that here the pilot-wave theory is providing an *explanation* for the process that is described in ordinary quantum theory as the collapse of the wave function. But whereas in ordinary quantum mechanics the collapse is an implausible, ad hoc exception to the usual dynamical rules, the transition of sub-system wave functions to appropriate eigenstates during suitable interactions is, in the pilot-wave theory, a *consequence* of the standard dynamical rules that apply all of the time.

7.5 Contextuality

In the earlier section, when we were thinking about the pilot-wave theory's account of the 2-slit experiment, we assumed that the visible "flash" on the detection screen occurs where the particle in fact hits the screen. We assumed, that is, that position measurements simply reveal the pre-existing positions of the particles. But the analogous thing does not appear to hold in the case of the energy measurement we discussed subsequently. The "measurement of the PIB's energy" had, to be sure, a definite outcome – E_n – but this in no way implied that the PIB somehow secretly had this particular amount of energy prior to the measurement interaction. Indeed, it's not even really clear what that would mean according to the pilot-wave theory: prior to the interaction with the measuring device, the PIB's wave function was a superposition of several energy eigenstates, and the particle had some definite position X within that wave; but there is simply nothing there that would allow us (or should make us feel the urge to) attribute some definite pre-measurement energy to the particle. It seems, instead, more reasonable to summarize the situation by saying that the PIB doesn't really have any particular energy prior to the measurement, although it does have one *after* the measurement.

Here is another example of how, in the pilot-wave theory, measurements do not necessarily just passively reveal pre-existing values. We mentioned at the beginning of this chapter that, for an electron in the ground state of Hydrogen or a particle-in-a-box potential, the (literal, pointlike) particle will be motionless. The same will be true for an electron in the ground state of a simple harmonic oscillator potential; let us analyze this case in some detail using some bits of mathematics that have already been worked out for other purposes.

Thus, consider an electron moving in one dimension which experiences the potential energy

$$V(x) = \frac{1}{2} m\omega^2 x^2. \tag{7.45}$$

The lowest-energy solution of the time-independent Schrödinger equation is a Gaussian wave function

$$\psi(x) = Ne^{-x^2/4\sigma^2} \tag{7.46}$$

where the width σ of the wave packet is related to the (classical angular) frequency ω of the oscillator and the mass m of the particle through

$$\sigma^2 = \frac{\hbar}{2m\omega}. \tag{7.47}$$

The energy eigenvalue of this state is $E = \frac{1}{2}\hbar\omega$.

If the electron is in this ground state, its wave function will be

$$\psi(x,t) = Ne^{-x^2/4\sigma^2} e^{-iEt/\hbar}. \tag{7.48}$$

The complex phase $S(x,t)$ depends on t only and so, like the earlier examples, Eq. (7.4) implies that the velocity of the (literal, pointlike) particle is zero, regardless of its precise location X within the Gaussian wave packet.

And this of course means that the *momentum* of the particle is zero as well, assuming that by "the momentum of the particle" we just mean its mass multiplied by its instantaneous velocity: $p = m\frac{dX}{dt}$. But this should be slightly troubling since, as discussed in Chap. 2, the generalized Born rule implies that a measurement of the momentum of the electron in this situation is exceedingly *unlikely* to yield the value $p = 0$. Recall in particular that the Gaussian wave function $\psi(x) = Ne^{-x^2/4\sigma^2}$ can be written as a linear combination of momentum eigenstates $\psi(x) = \int \phi(k)\frac{e^{ikx}}{\sqrt{2\pi}}dk$ with

$$\phi(k) = \sqrt{2}N\sigma\, e^{-k^2\sigma^2}. \tag{7.49}$$

This, according to the generalized Born rule, implies that the probability for a momentum measurement to yield a value between p and $p + dp$ is

$$\begin{aligned} P(p)\,dp &= P(k)\,dk \\ &= |\phi(k)|^2 dk \\ &= 2N^2\sigma^2\, e^{-2k^2\sigma^2}\, dk \\ &= \frac{2N^2\sigma^2}{\hbar}\, e^{-2p^2\sigma^2/\hbar^2}\, dp \end{aligned} \tag{7.50}$$

where we have used $p = \hbar k$ to relate the wave number k to the momentum p.

It is thus clear that, if the pilot-wave theory is going to be able to reproduce the usual quantum statistical predictions, it cannot be that momentum measurements simply reveal the pre-existing momentum! And, of course, it turns out that they do not. To understand in detail what the theory does say, about how such measurements will come out, we just need to analyze the measurement procedure in detail, using the theory.

This particular example lends itself well to imagining a so-called "time-of-flight" procedure for measuring the momentum. The idea here is that, to determine the momentum of the electron, one could "turn off" the potential energy $V(x)$ which confines the electron to the vicinity of the origin, let the particle fly freely away from the origin for a long time, observe its *position*, and then infer what the momentum must have been to allow it to arrive at that position.

We have already worked through all of the mathematics required to analyze this type of momentum measurement. For example, we saw in Sect. 7.3 that, for a free particle whose wave function is, at $t = 0$, $\psi(x, 0) = N\, e^{-x^2/4\sigma^2}$, the particle trajectories are given by Eq. (7.30). Since our momentum measurement procedure involves letting the particles fly freely for a very long time, it is sufficient to take the large t limit in which

$$X(t) \approx X_0 \frac{\hbar t}{2m\sigma^2}. \tag{7.51}$$

If the particle is observed at $X(t)$ at time t, we will infer that its velocity has been $v = X(t)/t$. Thus, according to the pilot-wave theory, a particle whose initial ($t = 0$) position was X_0 will produce a measured momentum value

$$p = mv = \frac{X_0 \hbar}{2\sigma^2}. \tag{7.52}$$

Qualitatively, one sees how this particular method of measuring the momentum of the electron yields non-zero values even though the electron's momentum was, just prior to the initiation of the measurement procedure, zero: turning off the confining potential energy changes the subsequent time-evolution of the electron's wave function, which in turn causes the particle to acquire a non-zero momentum! (Or, at least, this is what happens if, as is overwhelmingly probable, the particle's initial position does not happen to be precisely $X_0 = 0$.) The outcome of the measurement does indeed in some sense come into existence as a result of the measurement intervention. But the process by which this occurs is clear and comprehensible and governed by the same quantum laws that (according to the pilot-wave theory) always apply.

It is easy to check also that the quantitative statistics work out correctly. Using Eq. (7.52) to relate the measured momentum value p to the initial position X_0 of the particle within the wave packet, we may assert that the probability for the momentum measurement to yield a value between p and $p + dp$ is equal to the probability that the initial position of the particle was in the range that would lead to those outcomes. But then we know how to express that probability in terms of the initial wave function. Putting these pieces together, we find that

$$\begin{aligned} P(p)\, dp &= P(X_0)\, dX_0 \\ &= |\psi(X_0, 0)|^2\, dX_0 \\ &= N^2\, e^{-X_0^2/2\sigma^2}\, dX_0 \\ &= \frac{2N^2\sigma^2}{\hbar}\, e^{-2p^2\sigma^2/\hbar^2}\, dp. \end{aligned} \tag{7.53}$$

This is precisely the same Gaussian distribution of p-values that we found, in Eq. (7.50), was predicted by the generalized Born rule of ordinary quantum mechanics. So the pilot-wave theory not only explains qualitatively how a particle whose pre-measurement momentum is zero can nevertheless be measured to have a non-zero momentum, but it precisely agrees with ordinary quantum mechanics about the precise statistical distribution of those measured values.

This example nicely illustrates the point that, for measurable quantities other than position, measurements according to the pilot-wave theory do not just passively real the pre-existing value of the quantity in question. This is part of what is meant by saying that, for the pilot-wave theory, properties like momentum (and energy and spin) are "contextual".

But this notion of "contextuality" goes a little bit deeper. It is not just that the result of a measurement of a certain property can in general be different from the pre-measurement value of that property. Rather, there may be no such meaningful thing as "the pre-measurement value of that property" at all! We have already suggested something along these lines in the case of the measurement of the energy of the PIB (whose wave function is initially a superposition of several energy eigenstates). A complete description of the pre-measurement state of the PIB consists, according to the pilot-wave theory, of the PIB wave function (a superposition of several energy eigenstates) and the position X of the associated particle. It is simply not clear how, from these ingredients, one would construct some specific energy value to attribute to the PIB as a "pre-measurement value".

But, you might object, the pilot-wave theory is deterministic! So surely the outcome of the energy measurement (i.e., the final position $Y(t)$ of the energy-measuring-apparatus pointer) is determined by, i.e., is some complicated function of, the initial states of the PIB and the pointer and the details (captured by the interaction Hamiltonian H_{int}) of their interaction. That is true, but does not affect the overall point. The heart of the matter is that the measurement outcome depends not only on the initial state of the measured system (and the initial state of the measuring apparatus) but also on details pertaining to the specific *way* the measurement is carried out.

Concretely, in our example of the measurement of the energy of the PIB, the outcome of the measurement will depend not only on the initial conditions – $\Psi(x, y, 0)$, $X(0)$, and $Y(0)$ – but also on the value of λ, which controls the "strength" of the PIB-Pointer interaction and so determines, for example, the amount of time it takes for the configuration space "islands" described around Fig. 7.5 to separate. From a purely dynamical point of view, it is hardly surprising that the configuration point $\{X(t), Y(t)\}$, which after all moves in some complicated and erratic way while the "islands" are still in the process of separating, can end up in a different "island" depending on the amount of time it takes for them to separate. But this means that different – and perfectly, equally legitimate – methods of "measuring the energy of the PIB" will yield different measurement outcomes, even if they are implemented on perfectly identical systems. Surely this demonstrates the complete pointlessness of trying to imagine that there is, according to the pilot-wave theory, some pre-existing

energy value which is revealed by (or even somehow affected by and then revealed by) the measurement procedure.

Bell has pointed out that any residual feeling of queasiness – about the fact that "measurements" do not, in general, merely reveal some pre-existing value for the quantity being measured – is almost certainly just a result of the connotations of the word "measurement". If, to you, the word "measurement" *means* "simply finding out something that was already definite" then it turns out that, according to the pilot-wave theory, (the procedures that are conventionally called) "position measurements" are indeed genuine measurements, whereas (the procedures that are conventionally called) "energy measurements" (and "momentum measurements" and "spin measurements"...) are not actually measurements at all. Perhaps, as Bell suggested, using a different word (like "experiment" instead of "measurement") would help us avoid inappropriate expectations. But there is nothing here that is actually problematic:

> the word ['measurement'] comes loaded with meaning from everyday life, meaning which is entirely inappropriate in the quantum context. When it is said that something is 'measured' it is difficult not to think of the result as referring to some pre-existing property of the object in question. [But t]his is to disregard Bohr's insistence that in quantum phenomena the apparatus as well as the system is essentially involved. If it were not so, how could we understand, for example, that measurement of a component of 'angular momentum' – in an arbitrarily chosen direction – yields one of a discrete set of values? When one forgets the role of the apparatus, as the word measurement makes all too likely, one despairs of ordinary logic – hence 'quantum logic'. When one remembers the role of the apparatus, ordinary logic is just fine [6].

For our purposes, all of this is important because it allows us to understand how, exactly, it is possible for the pilot-wave theory to exist, and work, in the face of the "no hidden variables" theorems that were mentioned back in Chap. 3. It seems that the (largely unacknowledged) linguistic connotations of the word "measurement" have contributed significantly to generations of physicists abandoning the hidden variables program and succumbing to the Copenhagen philosophy (or entertaining even more radical proposals such as the abandoning of the laws of logic). In particular, the conventional use of the word "measurement" (to describe experiments which output a value for the position, momentum, energy, spin, etc., of a particle) has led people to believe that it is reasonable to insist that any "hidden variable" account of such processes would have to attribute definite pre-measurement values that are simply revealed by the measurement, for all such quantities. But the "no hidden variables" theorems prove that "hidden variable" theories *of that sort* are impossible.

The "no hidden variables" theorems, that is, invariably apply only to "non-contextual" hidden variables theories. That is why those theorems do not in an any sense rule out the pilot-wave theory. But more importantly, the pilot-wave theory shows rather clearly that "contextuality" is in no way contrived or problematic, but is instead a completely straightforward consequence of the theory's very natural dynamical postulates. One just needs to take seriously the idea (which is inherent in understanding the measurement problem as a problem) that what a theory says about "measurements" should be extracted from (rather than awkwardly appended to) the theory's fundamental dynamical postulates.